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Mutual Causality and the Generation of Biological Control Systems

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This paper gives a semantic description of two operations in neural systems which may allow a formal bridge from theoretical neuroscience to quantum logic. This research is part of an ongoing effort to develop foundational mathematics in neurobiology to give insight into sensorimotor coordination, that is, the neural control of movement that is coordinated with the senses. Using a formalism developed for analyzing sensorimotor coordination, *conditional dynamics*, control structures for several sensorimotor behaviors—some of clinical significance—have been proposed. A particular form, the *dyad*, has arisen repeatedly, giving rise to the question: Can any sensorimotor control system for everyday complex movements be generated from a primitive control system? This paper opens the discussion by proposing a primitive control system and two operations, *budding* and *coupling*, for building more complicated control structures. Mathematical (syntactic) and empirical questions arising from this approach are discussed.

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1. INTRODUCTION

Everyday movements, such as walking, involve distinct conditions determined both by different external forces and by different neural control. For example, each leg alternates between swinging forward, unweighted, and then, after heelstrike, bearing the weight of the body. The intricate sensory and motor coordination with the rest of the body includes distinct movements made when the body is threatened with falling: fast postural adjustments. The relationships among these distinct conditions form a logical structure of walking. Skipping, for example, has a different logical structure, including obligatory loft phases. Everyday activities involve switching among movements with different logical structures. This multiplicity of logical structures

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gives neural control a kinship with quantum logic, which formalizes the system of logical systems that occur in quantum phenomena (McCollum, 2000; Randall and Foulis, 1983).

The multiplicity of logical structures includes not only motor structures, as in the distinction between walking and skipping, but also sensory structures. For example, walking and bicycling differ not only in movements, but also in the ways the senses are used and coordinated with movements. The multiplicity of logical structures occurs throughout neural systems, from the level of sensorimotor behavior to that of neural mappings. Although the logical structure of a neural mapping or sensorimotor coordination is finely matched to its evolution and physical function, it cannot in general be deduced from physics (Rota, 1986). This paper is part of a research program which includes attempts to model logical structures in neurobiology directly, using concepts and mathematics tailored to each neural system (Fig. 1). Physicists and mathematicians have a tradition of developing foundational mathematics; in our society, physicists and mathematicians are the only people expected to do so (Wertheim, 1995). For this reason, physicists and mathematicians have an important role to play in nurturing foundational mathematics in biology.

Experimental neuroscientists have characterized the dynamics of single movements such as forearm movements in a fixed plane (Viviani and Terzuolo, 1981; Hollerbach and Flash, 1982; Levin *et al.*, 1995), smooth eye movements (de'Sperati and Viviani, 1997), and fast jumps of eye position

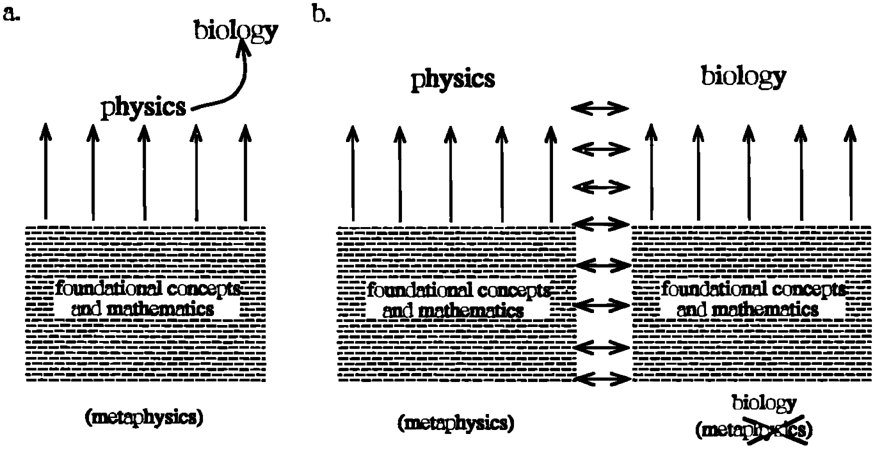

Fig. 1. Diagram of the relationships among scientific domains. (a) Biology as a derivative branch of physics. In this case, biology has no foundational concepts and foundational mathematics of its own, but must inherit them from physics. (b) Biology as separate but interacting with physics. When logical systems in biology are modeled directly, biology has its own foundational concepts and foundational mathematics.

called "saccades" (Viviani *et al.*, 1977; Feldman, 1981). Invariants of motion followed in complex movements such as pointing with the arm (Soechting and Lacquaniti, 1981), walking (Borghese *et al.*, 1996), and free arm movements (Soechting *et al.*, 1986; Soechting and Terzuolo, 1987) have also been identified. In everyday life, these dynamics and invariants are imposed in free-flowing sequences and combinations. Unfortunately, the ability to compose and perform such complex movements can be prevented by a neurological disorder. Therapies for such patients may be developed from insights gained by analyzing the sensorimotor control of complex movements. In order to analyze the sensorimotor control of multijoint, multiphase movements, a formalism, *conditional dynamics*, has been developed to allow analysis of observed behaviors in which (1) the behavior is caused by the dynamics of multiple, distinct agencies, such as gravity and various neural centers (parts of nervous systems), (2) the underlying dynamics of neural centers are often not available for observation, and (3) the behavior is conditioned by abrupt changes in the dynamics of participating agencies, whether neural or external to the nervous system, or in their relations to each other (McCollum, 1994a, b, 1998a). Conditional dynamics allows the modeling of sensorimotor coordination as structures of action or control, found or designed to be viable.

Conditions are expressed as localizations of action, or *regions*, for example, the localization of leg motion to the cycle typical of walking. As in quantum mechanics, the theory sought is comprehensive, but not complete (Finkelstein, 1997). It is important in biology to specify motion only partially, to allow for the variations within which the motion is functionally equivalent (McCollum, 1998a). For example, the stance phase of walking—when the leg bears the weight of the body—varies with each step cycle (Fig 2a). Many aspects of stance phase are not specified here, including muscle activity, gaze position, and speed; these aspects exist, have ranges of values, and could be specified. Inclusion is the first of two algebraic relations between regions. Included within the stance phase is the end of stance phase, when the leg is toward the back (away from the direction of progression) (Fig. 2b). The end of stance phase triggers a change in neural and sensorimotor dynamics, besides in body mechanics, leading to swing phase, in which the leg swings forward (Fig. 2c). "Leading to" is formalized as *contiguity*, a second algebraic relation between regions and diagrammed as a dashed arrow (McCollum, 1994a, 1998a).

From the point of view of one leg, there is an alternation between stance and swing phases, which can be formalized as a *dyad* consisting of two states, stance and swing, along with two trigger regions through which each leads to the other (Fig. 3) (McCollum, 1998b). In learning to walk, a child differentiates these two phases—"differentiates" in the biological sense of separating into distinct functional regions two parts of a physical continuum (Maturana

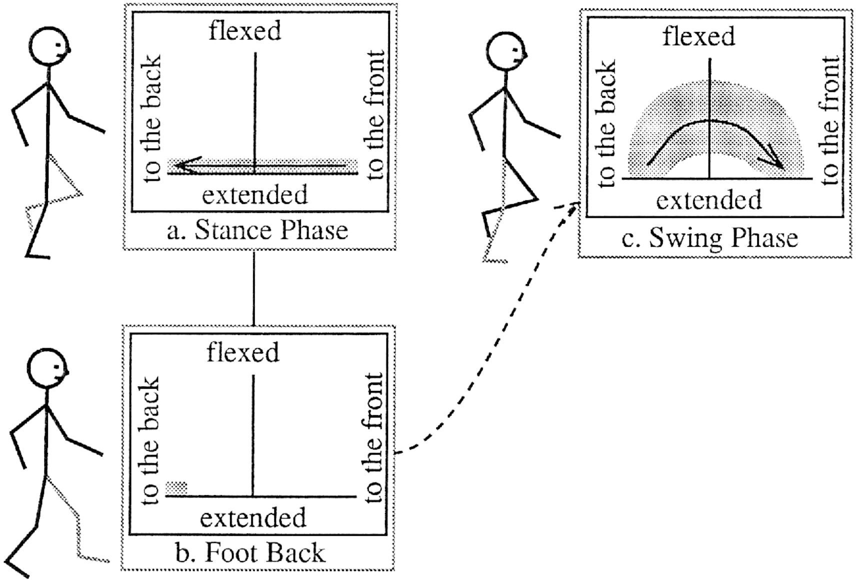

Fig. 2. Transition from stance phase—when the leg is bearing weight—to swing phase, for one leg. Boxes represent regions, the solid line denotes inclusion, and the dashed arrow denotes contiguity.

and Varela, 1987; Varela, 1975). By differentiating stance phase from swing phase, a child can learn to use the different physical properties of each (McCollum, *et al.*, 1995) and to avoid the more ambiguous properties, such as friction, that arise in intermediate zones. A dyad formalizes this structure.

Dyads have arisen in modeling sensorimotor control systems (McCollum, 1999) and from explorations of conflicting control (McCollum, 1998b). These occurrences, plus their resonance with previous biological literature

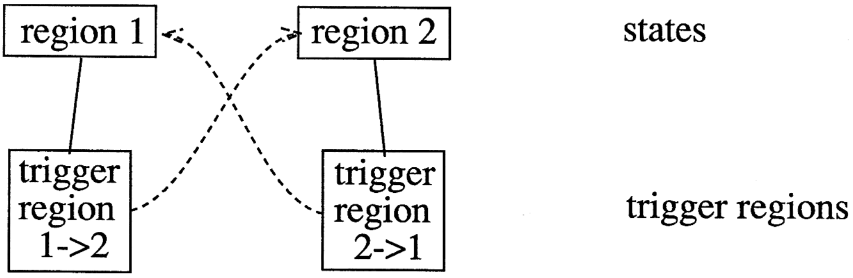

Fig. 3. Dyad. In Fig. 2, the trigger region is b. For a sleeping–waking dyad, the trigger region leading from waking to sleeping is the preparation for sleeping that occurs when one is awake, and vice versa.

(Maturana and Varela, 1987; Varela, 1975), led me to wonder whether all biological control systems might be generated by differentiation and by combining differentiated controls. For example, half-dyads (Fig. 2) can be combined to make dyads or more complicated control systems. However, control systems generated from half-dyads would need to be subject to further constraints in order to function as smoothly and precisely as normal sensorimotor coordination (McCollum *et al.*, 1996). To be consistent, the control system would need to allow each state to be both entered and left. To be unambiguous, each trigger region needs to lead to only one state. The control system needs to be stable, complete, and viable in ways that depend on the universe of contingencies, that is, on the range of situations in which the control system is used.

Using the dyad itself as the primitive control system from which to generate more complicated control systems avoids these issues, because it is already consistent and unambiguous. This paper presents two operations that maintain those properties while elaborating control systems.

2. BUDDING

Once a dyad is established as a control of behavior, such as sleeping and waking, states can be differentiated within each state. Within sleeping, states are distinguished observationally by means of the electroencephalogram (EEG) and eye movements (Fig. 4). Although each particular night's (or day's) sleep varies, there is a standard sequence of sleep states. There are four distinctive EEG patterns, labeled EEG 1 through EEG 4, distinguished by decreasing EEG frequency and increasing EEG amplitude. EEG 1 can

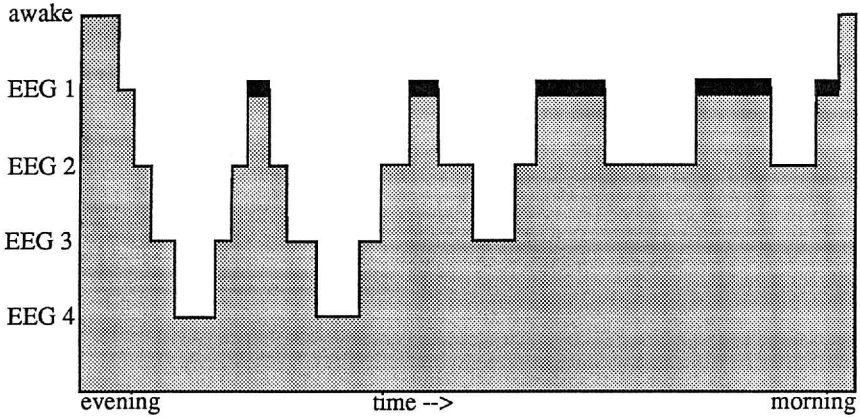

Fig. 4. Diagram of a typical sequence of sleep states through the night. REM sleep is denoted by heavy bars during EEG 1. Modified from Kandel and Schwartz (1985).

include dream sleep, which is identified observationally by rapid eye movements, and is therefore called REM sleep. An investigation of the development of the sleep sequence—which states actually differentiate and in which order—is beyond the scope of this paper, as is a thorough description of sleep phenomena. Instead, this section will present an operation by which a control structure can be expanded to describe transitions between substates of a state.

Given a dyad, one of the states in the dyad can differentiate into two included substates, connected by a dyad structure. For example, sleeping is divided into stage 1 and stages 1+ (i.e., 2–4) (Fig. 5a). This control structure shows the ability of the system to alternate arbitrarily between stage 1 and states 1+ during sleep. Stage 1 is divided into REM and non-REM (Fig. 5b). By thus successively "budding" dyads, a control structure is constructed that reproduces the passage between successive stages (Fig. 5c). It is left for further research to find out exactly which of the variations allowed by this control structure occur.

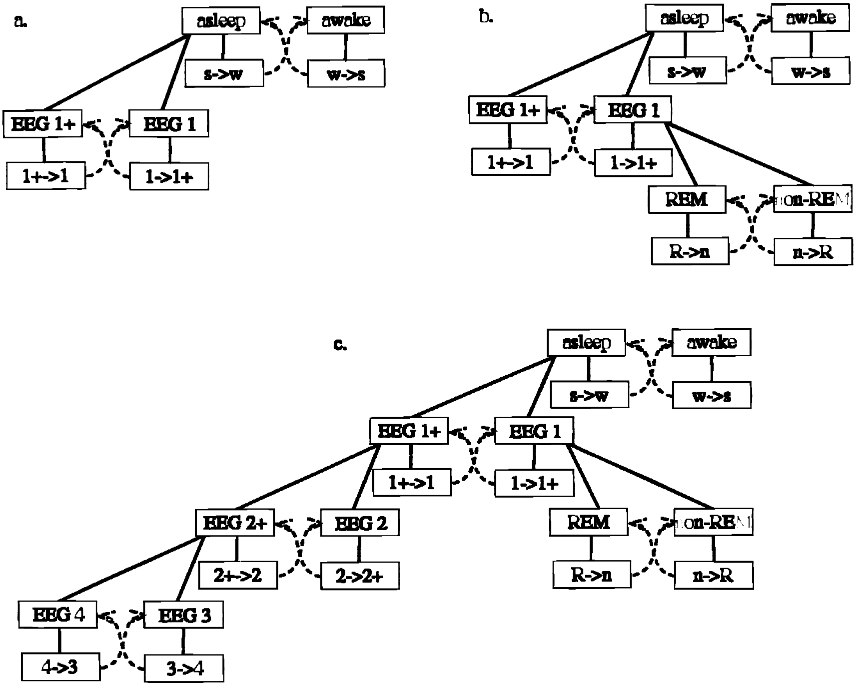

Fig. 5. Generation of a sleep control structure by budding. (a) Differentiation of "asleep" into high-frequency (EEG 1) and lower frequency (EEG 1+). (b) Budding of "EEG 1" into dreaming (REM) and nondreaming (non-REM). (c) Complete control structure giving patterns like Fig. 4, produced from b by two more budding steps.

One variation, which has been investigated extensively, is the change in sleep pattern over the period of sleep (Kandel and Schwartz, 1985). Among other factors, biochemical agents change in concentration during the course of sleep. They may govern the triggers that lead to changes of state (Fig. 6). Such a mechanism may account for the reduction in the deeper (higher number) sleep stages and the increase in REM sleep. For example, changes in biochemical concentration over the course of sleep may increase the likelihood of entering trigger regions leading to lighter (lower number) sleep stages and leading to REM sleep, relative to those leading to deeper sleep stages and to those leading out of REM sleep. A traditional way to express transition probabilities is in terms of a continuous manifold, often a potential. Such a potential could be considered to change as the period of sleep progresses, at first favoring sleep and later favoring waking. To model all the sleep stages and the transitions between as one dynamical manifold, rather than the conditional dynamical version (Figs. 5 and 6), would require more forethought and would be more difficult to vary for particular situations and individuals.

However, it is important to include the known continuous dynamics, both within and impinging upon biological systems, in conditional dynamical models. Some dynamics governs biological control—for example, the bio-

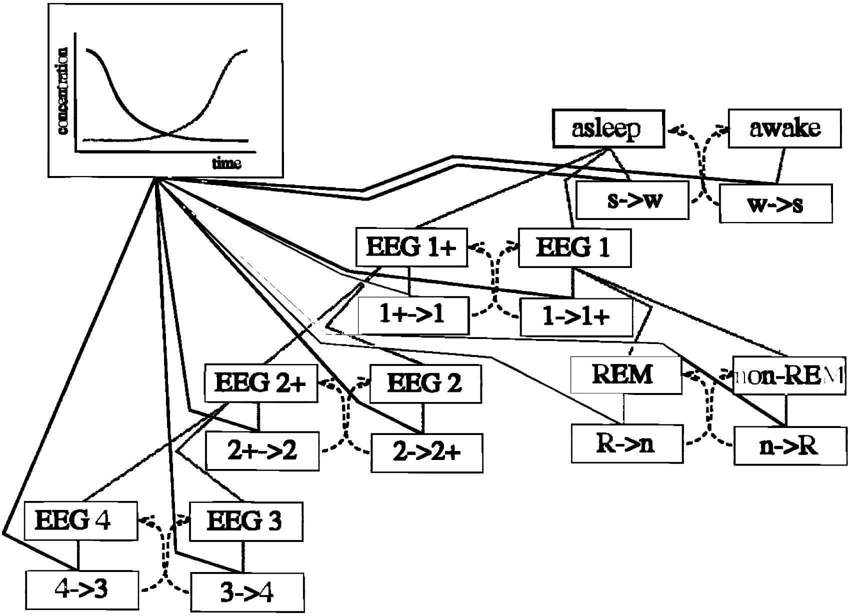

Fig. 6. Chemical concentrations governing trigger regions. Different patterns are represented by one concentration decreasing while the other increases.

chemical concentrations discussed with the sleep example (Fig. 6) and the dynamics of physical bodies within a gravitational field. It has been proposed that each individual's nervous system contains a metric tensor, determined genetically, which governs individual sensorimotor style (Pellionisz and Llinás, 1979). While the details of this idea have not so far been confirmed, there are factors governing large portions of an individual's sensorimotor style. Other dynamics occurs within a region—for example, the dynamics of an octopus's tentacle when reaching out (Gutfreund *et al.*, 1996) or the dynamics of eye movements, which have similarities across many species.

Mathematical questions to be addressed include: What is the right way to formalize the relationship between this algebraic approach to conditional dynamics and the dynamical systems approach? When is it legitimate to lump a complex—such as walking—of distinct dynamical systems into one?

3. CONTINGENT RELATIONS BETWEEN REGIONS

Regions are not necessarily distinct. Whether they are distinct can depend on contingencies: a stranger can turn out to be your uncle. The same region can be used in different movements, for example, the same leg position can occur in walking or in skipping. Also, whether regions are distinct can depend on neural factors (including biochemical) in producing regions or sensory factors in distinguishing them, so that two regions may gradually become one. This is a subject for further research, especially empirical: In what ways do neural systems give up distinctions once made?

Consider the dyad controlling one leg's movements through the step cycle of walking. As a child learns to skip, the stance phase differentiates into a liftoff phase and a landing phase (Fig. 7a). This is essentially a hop movement, so that liftoff and landing could be executed an arbitrary number of times. However, in the context of moving forward at skipping speed, only one hop and loft is executed per cycle per leg. This contingency has consequences that change the apparent form of the control structure. The transition from stance to swing always occurs upon landing, because the leg is far enough back then; that is, the trigger region is included in "landing." Because liftoff only occurs once, it occurs at the beginning of stance phase; that is, the transition from the forward position of swing is essentially directly to "liftoff." The "stance" region may be shown, but may not be necessary part of the control structure (Fig. 7b). In this way, contingency transforms a control structure based on dyads into a threesome.

Besides the mathematical questions that arise—exactly how to formalize these contingent restructurings—there are also empirical questions. Is this process a faithful description of the development of a control structure?

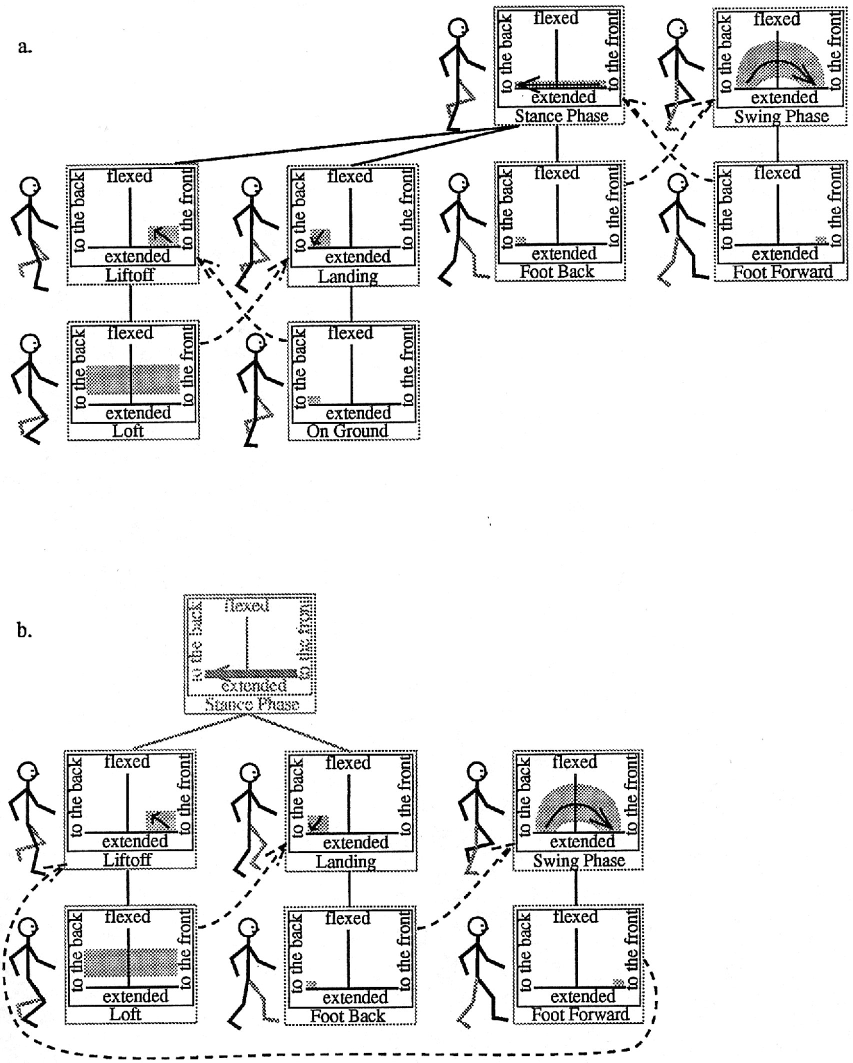

Fig. 7. Development of a triphasic control structure from dyads. (a) Stance phase budding into a hop with liftoff and landing phases. (b) Triphasic result of functional conflation of regions.

Do sensorimotor control and neural mappings undergo such differentiations and amalgamations?

Observations of patients indicate that regions are eliminated when they no longer function for the patient, for example, when the patient has lost sensory functions that certain behaviors depend on. The result may be that the control structure is no longer stable and viable in the patient's environment.

For example, there is a group of patients with balance disorders, who have an apparent control structure as shown in Fig. 8. The two sensory coordination modes refer to neural interrelations among the orientation senses, which are the visual, somatosensory, and vestibular systems. Different sensory coordination modes are appropriate for different circumstances, for example, on solid ground versus on a water bed. This particular group of patients displays only two sensory coordination modes (with eyes open in light), with two trigger regions, visual surround moving and floor not moving. In many circumstances, this control structure is effective. However, the trigger regions have a nonempty intersection, for example, standing on a sturdy (nonmoving) tree branch with the wind blowing the leaves. In such circumstances, these patients become motion sick, perhaps because of a control reverberation, an inability to settle in one sensory coordination mode.

A similar mechanism may explain space-sickness. Suppose people on earth develop sensorimotor control structures with contingently noninter-

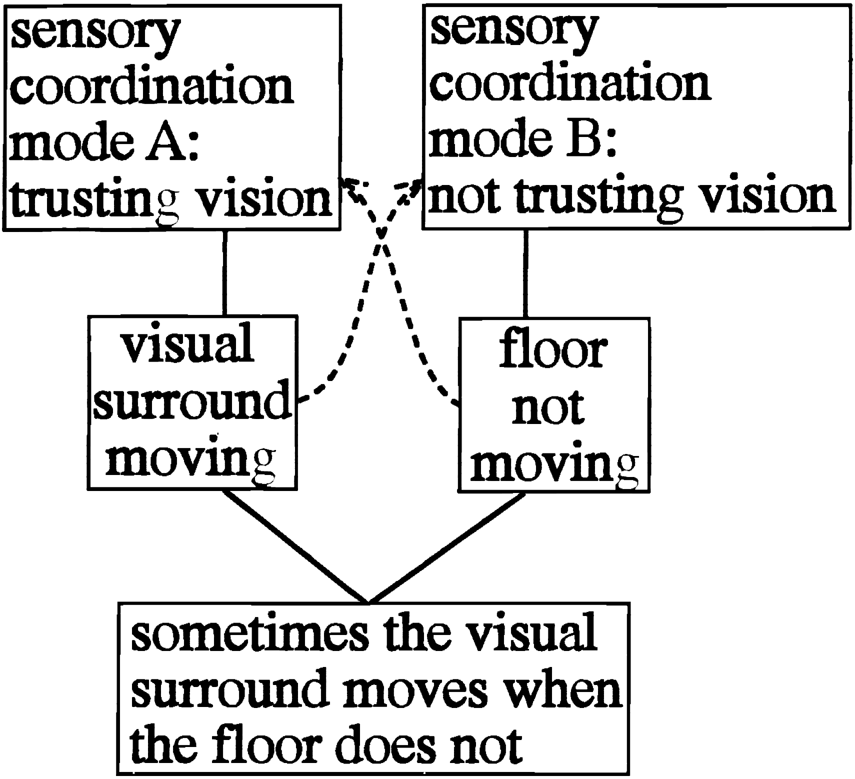

Fig. 8. Trigger regions with nonempty intersection.

secting trigger regions. That is, on earth, no intersection between the trigger regions is ever encountered, even though an intersection is physically possible. The intersection may then be encountered in free-fall, leading to space-sickness and the necessity to reconfigure the sensorimotor control structure, or “adapt.” Sailors similarly adapt to sea conditions. Some sailors are “dually adapted,” in the sense that they have separate sensorimotor control structures for sea and land, and do not need to go through an adaptation period when switching between sea and land (Cohen, 1996; Helling and Westhofen, 1994), just as many people are dually adapted to walking and bicycling. For space programs, dually adapted astronauts would be advantageous, both to save time at the beginning of the flight and for safety on landing.

In considering these practical questions, mathematical questions arise, including: What are the possible relations between regions (such as intersection and inclusion), especially trigger regions, and what effect does that have on the functioning of the control system? Suppose that trigger regions are found that are non-intersecting on earth but intersecting in free-fall. For that situation to cause control reverberation (and motion sickness), does the discrete structure have to be that shown in Fig. 8?

4. COUPLING

By some means, whether distant differentiation or origination in separate physiological systems, controls may come together in a nervous system as separate controls. This is the apparent situation in the control of fast postural adjustments. A range of movements may be used, including shifting weight from foot to foot (Nashner and McCollum, 1985; Runge *et al.*, 1998). Even for small perturbations in the forward–backward direction, a range of combinations of hip and ankle torques may be used (Fig. 9a), resulting in a range of combinations of torque and shear force being exerted on the support surface. Surface conditions, such as compliance or slipperiness, may induce the nervous system to differentiate the range into distinct movements, a rigid rotation about the ankle or a hip movement with secondary ankle movement (Fig. 9b) (McCollum *et al.*, 1985; Horak and Nashner, 1986). Similar environmental considerations lead to changes in sensorimotor coordination modes. Control of sensory and motor states combine in the control of fast postural adjustments.

Controls that do not affect each other may combine without coupling or coordinating. However, the combinations of sensory signals caused by postural movements differ. For example, a forward rotation about the ankles causes the visual field to stream backward while the somatosensory system senses a flexion of the ankle. In contrast, in a hip movement for which the visual field streams backward, the ankles extend (Fig. 10). Thus, in order to

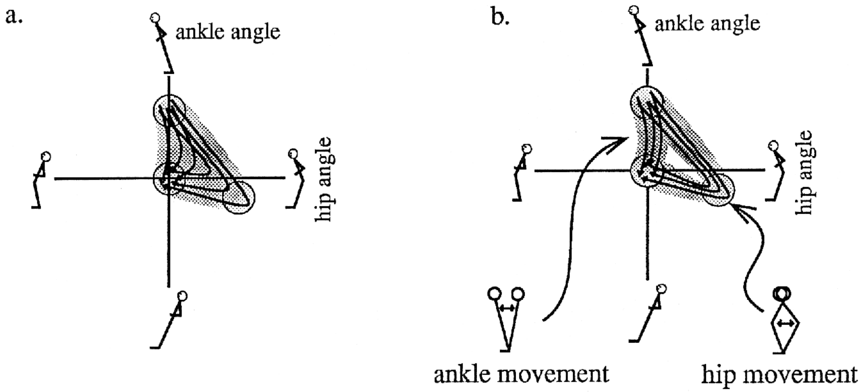

Fig. 9. Regions of postural movements. (a) Full continuum. (b) Separated ankle movement and hip movement continua.

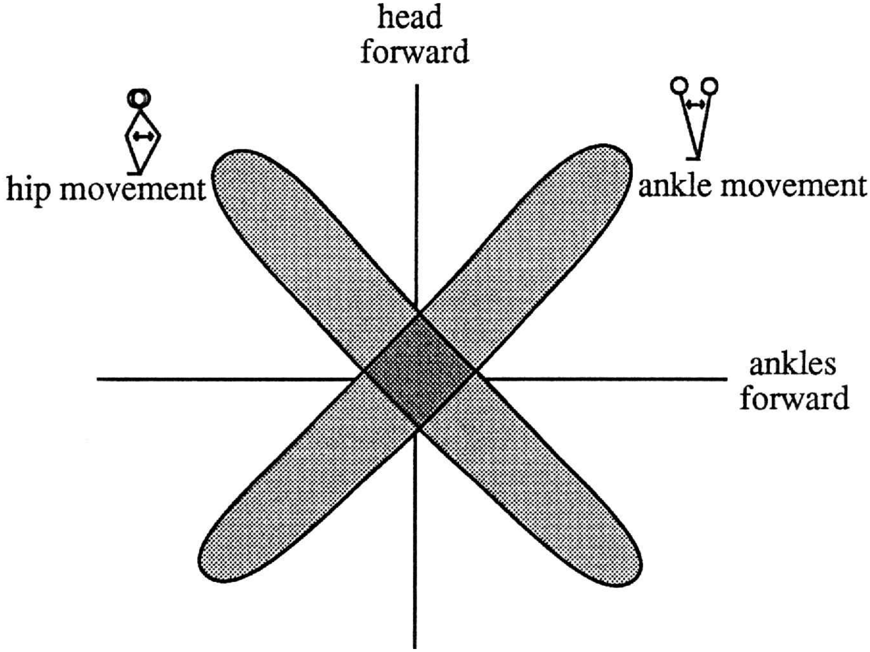

Fig. 10. Distinct combinations of sensory responses to distinct postural movements.

signal whether a movement is voluntary (not a perturbation), the sensory system must be coupled to the motor system.

For simplicity, consider exactly two sensory states and exactly two motor states, that is, one sensory and one motor dyad (Fig. 11a). In order for the transitions within the sensory (motor) dyad to be governed by the motor (sensory) state, there must be separate trigger regions, depending on the motor (sensory) state (Fig. 11b).

This product between dyads bears some similarity to the tensor product in quantum logic, which has been used to represent coupling in physical systems (Aerts and Daubechies, 1978). Formalization and relationships to other mathematical structures are part of ongoing research.

5. FURTHER UNANSWERED QUESTIONS

Investigation of the generation of biological control structures may lead to insights into complex control structures, such as for piano playing (Engle *et al.*, 1997), and to insights that will help patients. The idea of generating biological control structures owes a major debt to Greechie spaces (Kalmbach, 1983). As a syntactic structure is developed to match the operations sketched here, it will be continually tested against a range of mathematical and empirical questions, including: What subset of the generated set of control structures is actually used by biological systems? How can that subset be specified or—better yet—generated? Is that how organisms do it? What is the range

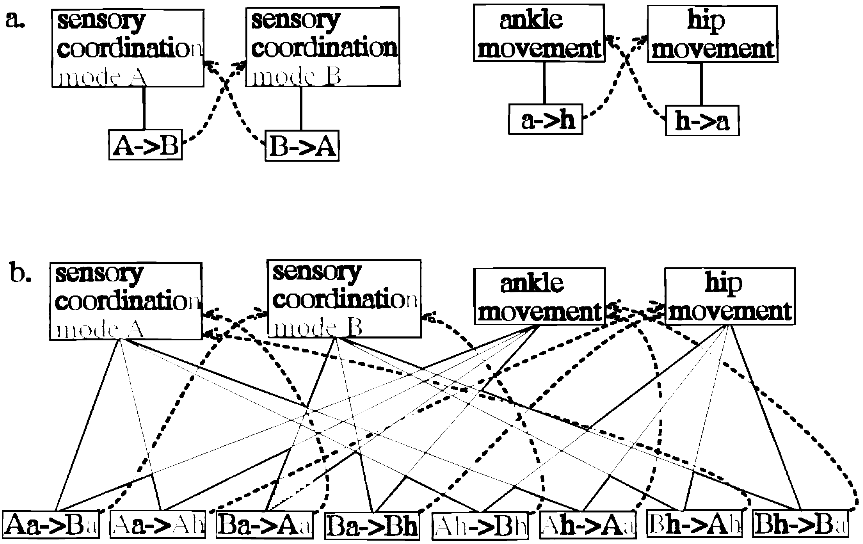

Fig. 11. Coupling. (a) Two dyads. (b) Two dyads coupled.

of biological control structures that belong to this subset? What is the range that do not?

Present research includes more cases in which control structures reverberate and the strategies that individuals use and do not use to tame that reverberation. The possibility of "miscoupling" in various ways is also being investigated to explain difficult experiences that patients have.

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